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THE PRECIPITATION DAY STATISTIC

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ABSTRACT

The first-order station annual totals of days with measurable precipitation are examined and found to be normally distributed. Isoline charts of annual means and standard deviations for the contiguous United States are presented, and quantitative guides used for isoline smoothing are described. The tendency of substation values toward lower means and greater dispersion is noted. The suggestion is made that more reliable probability statements of measurable precipitation occurrence can be obtained through use of the mean and standard deviation charts than from using substation data.

1. INTRODUCTION

The purpose of this paper is to develop charts that show the means and deviations of the annual number of days with measurable precipitation for the contiguous United States. In earlier work on the problem attempts have been made to use data from the substation or cooperative observer network that the U.S. Weather Bureau has operated throughout its existence. This network consists, in the main, of unpaid observers equipped with non-recording rain gages and, sometimes, other meteorological instruments. Over 11,000 such stations, manned by public-spirited citizens, presently provide supplementary weather information of incalculable value to the nation. However, the earlier work using data from this dense network revealed a bias in the substation precipitation day statistic.

Thirty years ago Armington [1] reported that counts of days with measurable precipitation at cooperative stations seemed consistently lower in all parts of the United States than comparable figures for first-order Weather Bureau stations. This was contrary to his expectations, and he could offer no explanation for it.

In 1953, during the course of other work the writer and two co-workers encountered the same bias, although it seems likely that others must have noticed it during the quarter-century interim. After the 1921–52 averages for several climatological items were plotted on base maps for South Carolina, it became apparent that the substation averages of days with 0.01 inch or more did not fit the rather smooth pattern formed by the same statistic for first-order stations. Further, the substation values very frequently disagreed, sometimes violently, with those printed for an earlier period in the 1930 edition of Bulletin W [2].

A few machine runs of punched data cards, which happened to be available at that time for several Missouri stations for a long record period, served to check this finding. The substation bias seemed to persist from the 0.01-inch or more threshold through the 0.02- and 0.03-inch thresholds, and began to weaken at about the 0.04- or 0.05-inch level. The final result of this little study [3] appears as table 1 in which the bias fade shows fairly clearly, except that the intermediate 0.02- through 0.04-inch values are not given. The rest of this earlier work is not presented here because it is no longer available. However, the substation bias will be discussed later in this paper.

These findings led to abandoning this statistic for substations, and, beginning in 1954, to substituting the 0.10-inch threshold for the 0.01-inch threshold for substations in current climatological publications. The 0.01-inch threshold was retained only for first-order stations.

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TABLE 1.—*Average annual number of days with precipitation (1918–1947)*

Station	<.01	≥.01	≥.05	≥.10	≥.15	≥.20	≥.25	≥.30	≥.35	≥.40	≥.45	≥.50
Illinois:												
Cairo*	252.5	112.5	84.7	71.2	61.2	53.7	47.8	41.9	38.3	34.7	31.3	27.7
Missouri:												
Jackson	268.0	97.0	84.7	73.7	64.9	58.8	52.7	46.8	42.7	39.4	35.6	32.7
Columbia*	255.0	110.0	81.9	68.3	58.8	51.6	45.6	40.4	36.0	32.3	29.1	25.9
Mexico	263.4	101.6	86.9	71.6	61.0	54.0	48.0	41.9	37.6	34.1	31.1	28.2
Macon	270.0	95.0	80.2	67.4	57.0	50.4	44.5	39.7	34.9	31.1	27.4	24.8
Marshall	282.3	82.7	75.8	64.8	55.7	50.6	45.0	41.1	35.8	32.0	28.9	26.6
Eldon	274.8	90.2	78.2	67.0	58.8	51.8	46.4	42.0	37.7	33.4	30.0	26.7
Kansas City*	261.5	103.5	75.0	60.5	51.6	45.6	40.5	37.0	33.2	29.6	26.4	23.8
Harrisonville	272.3	92.7	76.3	64.2	56.2	49.5	43.4	39.2	35.3	31.4	28.0	25.8

*First-order stations.

2. GAUSSIAN APPROXIMATION

In approaching this problem more recently, the writer postulated a Gaussian distribution fit to the unbiased annual total days with 0.01 inch or more of precipitation for any one station. The usual curve fitting methods were employed and the not unexpected results showed the normal curve to be an excellent approximation to first-order station data. Several substations were included in the tests as a matter of curiosity, and, although it was not expected, their data proved normally distributed also; the means, of course, were low, but this did not seem to disturb the Gaussian fit.

For this paper, a first-order station network of 169 stations was selected, and the individual annual 1929–58 total days with 0.01 inch or more of precipitation were extracted to provide a uniform 30-year record for all but four stations: Alamosa, Colo., 1933–58; Blue Canyon, Calif., 1930–58; Butte, Mont., 1929–30, 1932–58; Astoria, Oreg., 1929–48, 1950–58. The mean, ¯, and standard deviation, s, were calculated for each station, and the results appear in table 2. The Kolmogorov-Smirnov goodness-of-fit test, described by Massey [4], was used to prove the Gaussian fit instead of the classical chi-square test devised by Karl Pearson about 60 years ago. Not only was the Kolmogorov-Smirnov test easier to apply in this case, but it is at least as powerful [4] as the chi-square while its use avoids some of the inherent weaknesses of the chi-square test [5] which arise from the arbitrary decisions necessary to its application. There are, of course, still other tests for normality available in addition to these two.

It seemed sufficient to test only one station in each State for normality. For all of these the fit to the normal was proved at the 0.20 significance level, which is excellent. This should be interpreted as meaning that the data for none of the tested stations deviated from a fit to the normal distribution by as much as the very small amount allowed, whereas it would have been permissible for up to 20 percent of the stations to exceed this severe deviation limit and still make the fit-to-normal assumption acceptable. Although it is not particularly important here, one can make the point that the fit is probably not quite that good since it was necessary to use estimates, i.e., the table 2 values, of the distribution parameters. This has the

TABLE 2.—*Station list, means* ($\bar{x}$) *and standard deviations* (s) *of annual days with 0.01 inch or more precipitation*

	$\bar{x}$	s
ALABAMA:		
Birmingham	117.9	12.30
Mobile	116.0	12.24
Montgomery	111.7	12.23
ARIZONA:		
Flagstaff	80.2	14.82
Phoenix	35.6	9.16
Prescott	66.1	13.84
Tucson	49.7	9.93
Yuma	16.6	6.60
ARKANSAS:		
Fort Smith	96.3	15.48
Little Rock	104.7	16.89
CALIFORNIA:		
Bakersfield	37.6	8.41
Blue Canyon	85.3	13.07
Eureka	117.9	16.65
Fresno	44.8	8.94
Los Angeles	37.5	9.30
Oakland	63.3	11.22
Sacramento	57.7	11.11
San Diego	44.7	10.86
San Francisco	67.1	13.03
COLORADO:		
Colorado Springs	82.3	13.12
Denver	85.6	13.34
Grand Junction	74.1	14.44
Pueblo	69.8	11.58
CONNECTICUT:		
Hartford	129.2	11.66
New Haven	130.2	11.59
DISTRICT OF COLUMBIA:		
Washington	119.8	12.10
FLORIDA:		
Apalachicola	105.7	14.03
Fort Myers	117.8	15.37
Jacksonville	119.0	11.87
Key West	111.6	12.91
Miami	131.7	16.38
Pensacola	110.1	12.83
Tampa	111.1	11.78
GEORGIA:		
Atlanta	115.5	12.06
Augusta	108.4	12.32
Macon	111.5	11.39
Savannah	108.9	13.11
IDAHO:		
Boise	88.0	14.21
Pocatello	94.0	13.59
ILLINOIS:		
Cairo	113.8	12.43
Chicago	120.3	10.12
Peoria	111.1	9.28
Springfield	111.4	9.63
INDIANA:		
Evansville	116.7	9.84
Fort Wayne	129.1	10.17
Indianapolis	125.3	11.88
IOWA:		
Des Moines	102.9	12.59
Dubuque	110.9	12.30
Sioux City	94.3	12.91
KANSAS:		
Concordia	87.8	13.43
Dodge City	76.5	12.32
Topeka	95.3	14.01
Wichita	86.4	17.58
KENTUCKY:		
Lexington	134.4	12.49
Louisville	119.9	11.40
LOUISIANA:		
New Orleans	119.0	12.89
Shreveport	97.8	13.93
MAINE:		
Portland	132.3	17.21
MASSACHUSETTS:		
Boston	126.7	14.34
Nantucket	127.1	9.68
MICHIGAN:		
Alpena	147.3	11.52
Detroit	132.5	10.81
Escanaba	128.0	10.79
Grand Rapids	137.3	12.59
Lansing	139.5	11.39
Marquette	158.0	11.79
S. Ste. Marie	164.0	11.96
MINNESOTA:		
Duluth	122.5	10.64
St. Paul	110.1	11.84
MISSISSIPPI:		
Jackson	105.7	14.68
Meridian	108.7	14.80
Vicksburg	104.5	13.01
MISSOURI:		
Columbia	106.5	12.94
Kansas City	99.6	13.09
St. Joseph	100.0	13.32
St. Louis	109.2	14.33
Springfield	107.9	13.16
MONTANA:		
Butte	105.2	18.24
Havre	89.9	10.79
Helena	97.0	10.21
Kalispell	124.3	15.95
Miles City	87.6	14.33
NEBRASKA:		
Grand Island	81.6	13.45
Lincoln	93.6	12.55
North Platte	82.7	11.85
Omaha	93.8	12.04
Valentine	88.7	15.43
NEVADA:		
Reno	50.0	10.77
NEW HAMPSHIRE:		
Concord	124.9	13.06
NEW JERSEY:		
Atlantic City	118.9	11.76
Newark	122.6	12.94
Trenton	122.5	11.73
NEW MEXICO:		
Albuquerque	57.9	13.81
Roswell	51.9	13.19
NEW YORK:		
Albany	135.2	12.81
Binghamton	157.4	13.59
Buffalo	161.8	10.99
New York	122.5	11.31
Rochester	160.0	12.36
Syracuse	172.2	12.62
NORTH CAROLINA:		
Asheville	128.9	12.26
Cape Hatteras	118.9	14.82
Charlotte	115.1	11.58
Greensboro	118.7	12.08
Raleigh	114.5	13.29
Wilmington	117.6	14.37
NORTH DAKOTA:		
Bismarck	92.2	12.61
Devils Lake	104.3	10.79
Fargo	101.9	11.27
Williston	89.4	11.82
OHIO:		
Cincinnati	129.0	13.35
Cleveland	150.8	13.88
Columbus	132.1	10.44
Dayton	127.0	13.03
Sandusky	137.2	11.53
Toledo	128.6	9.95
OKLAHOMA:		
Oklahoma City	82.1	14.37
Tulsa	90.3	14.15
OREGON:		
Astoria	179.5	18.53
Eugene	144.2	18.33
Medford	100.5	13.05
Portland	150.6	18.21
Roseburg	131.0	15.83
PENNSYLVANIA:		
Erie	155.3	14.30
Harrisburg	124.6	12.36
Philadelphia	119.8	11.74
Pittsburgh	150.7	15.05
Reading	124.8	13.76
Scranton	135.2	12.59
RHODE ISLAND:		
Block Island	121.3	11.36
Providence	121.4	13.18
SOUTH CAROLINA:		
Charleston	107.3	14.43
Columbia	110.2	11.02
SOUTH DAKOTA:		
Huron	91.8	10.82
Rapid City	92.9	12.30

TABLE 2.—*Station list, means* ($\bar{x}$) *and standard deviations* (s) *of annual days with 0.01 inch or more precipitation*—Continued

	$\bar{x}$	s		$\bar{x}$	s
TENNESSEE:			VIRGINIA:		
Chattanooga	121.0	12.96	Lynchburg	124.7	18.16
Knoxville	127.2	12.78	Norfolk	119.8	12.59
Memphis	104.5	14.93	Richmond	118.7	13.39
Nashville	118.5	11.02			
			WASHINGTON:		
TEXAS:			Seattle	150.3	18.14
Abilene	65.2	14.32	Spokane	110.1	17.37
Amarillo	67.2	13.26	Tatoosh Island	197.5	17.45
Austin	84.4	13.50	Walla Walla	106.5	14.86
Brownsville	79.6	15.30	Yakima	68.0	13.01
Corpus Christi	78.1	15.77			
Dallas	79.0	14.81	WEST VIRGINIA:		
Del Rio	60.2	15.25	Elkins	168.5	17.31
El Paso	46.4	11.39			
Fort Worth	78.2	13.77	WISCONSIN:		
Galveston	94.3	11.73	Green Bay	116.1	11.05
Houston	104.7	12.72	LaCrosse	112.8	11.60
San Antonio	82.7	13.52	Madison	116.7	11.66
			Milwaukee	115.8	9.82
UTAH:					
Salt Lake City	90.0	11.57	WYOMING:		
			Cheyenne	101.4	12.87
VERMONT:			Lander	69.7	11.58
Burlington	149.9	12.68	Sheridan	104.0	12.80

effect of reducing further the already narrow deviation limit although the magnitude of this effect is not known [4].

3. CONSTRUCTION OF CHARTS

The station means and standard deviations (table 2) were plotted on charts in order that the smoothed isoline analyses could be made which appear here as figures 1 and 2. Naturally, any analysis of this sort is somewhat subjective, but the problem was made easier by the rather conservative nature of both statistics; i.e., they do not vary as rapidly with horizontal or vertical distance as do many other climatological elements such as total rainfall, temperature, and relative humidity, to name a few. Actually, little isoline smoothing was required, and that which was performed was done with recognition of the sampling error present in the calculated means and standard deviations themselves, and with attention to the major topographic features. The latter practice is familiar to climatologists, but the former may require some explanation.

If a different record period had been used, slightly different values would have appeared in table 2; this would have been true whether another 30-year period had been selected or whether a longer or shorter period had been used. Therefore, the table 2 values are only point estimates of the true values which are always unknown in climatology.

It became desirable, therefore, to obtain some quantitative measure of the possible departures of the means, $\bar{x}$, and standard deviations, s, of table 2 from their true or theoretical counterparts, μ and σ, respectively. This was done by constructing confidence intervals within which the true values must lie a desired portion of the time; i.e., in repeated sampling, a predetermined percentage of the several sets of intervals will contain the true means or standard deviations. This method provided additional isoline smoothing guides which were used wherever there were isoline sinuosities that could not be explained on the basis of topographic influences. The 90 percent confidence interval was selected here, and it was assumed there were no trends in the data.

In calculating the 90 percent confidence intervals for the means in this paper, use was made of the probability statement.

$$P[|\bar{x}-1.64\, s/\sqrt{n}|<\mu]=0.90 \tag{1}$$

where n is the number of years of record (30 in this case), 1.64 is selected from any normal probability table such as [6], and s, $\bar{x}$, and μ are as before. Theoretically, the true standard error (i.e., standard deviation) of the mean $\sigma/\sqrt{n}$ should be used instead of $s/\sqrt{n}$, but the fairly large size of n permits the substitution without significant error here. The correct method, and the one to be used when n is much smaller, makes use of the statistic $t=(\bar{x}-\mu)\sqrt{n}/s$ in which σ does not appear and which has Student's distribution with $n-1$ degrees of freedom. Then the probability statement

$$P[|(\bar{x}-\mu)\sqrt{n}/s|<t]=0.90$$

gives rise to the 90 percent confidence interval $\bar{x}-0.31s<\mu<\bar{x}+0.31s$ after the value of $t=1.699$ is obtained from a table of Student's distribution. The difference here, as can be seen by referring to probability statement (2) below, is insignificant. If the variances are not calculated in their unbiased form $s^2=\Sigma(x_i-\bar{x})^2/(n-1)$ as they are here, $\sqrt{n-1}$ should be inserted in the probability statement above in lieu of $\sqrt{n}$.

After substitution of the proper values in the probability statement (1), the 90 percent confidence interval of the mean becomes

$$\bar{x}-0.30s<\mu<\bar{x}+0.30s \tag{2}$$

Whenever there were isoline irregularities that could be explained on no other basis, inequality (2) was used and the line moved not more than $0.3s$ (which could be interpolated from figure 2 with accuracy sufficient for the purpose) with considerable assurance that no appreciable error was introduced into the analysis thereby.

In order to find the 90 percent confidence intervals for the standard deviation, use was made of the quantity $(n-1)s^2/\sigma^2$ which has a chi-square distribution with $n-1$ degrees of freedom. This form is used in order to be consistent, since the variances were calculated in their unbiased form $s^2=\Sigma(x_i-\bar{x})^2/(n-1)$ as preferred by many experimentalists. Most mathematical statisticians (c.f. [7]) prefer for ease of handling and for class instruction, variances expressed as $\Sigma(x_i-\bar{x})/n$, in which case the above quantity would become ns^2/σ^2. In most cases, the difference is unimportant, as is the case here, particularly when n is fairly large. Then, the assumption was made [8] that the probability is 0.90 that

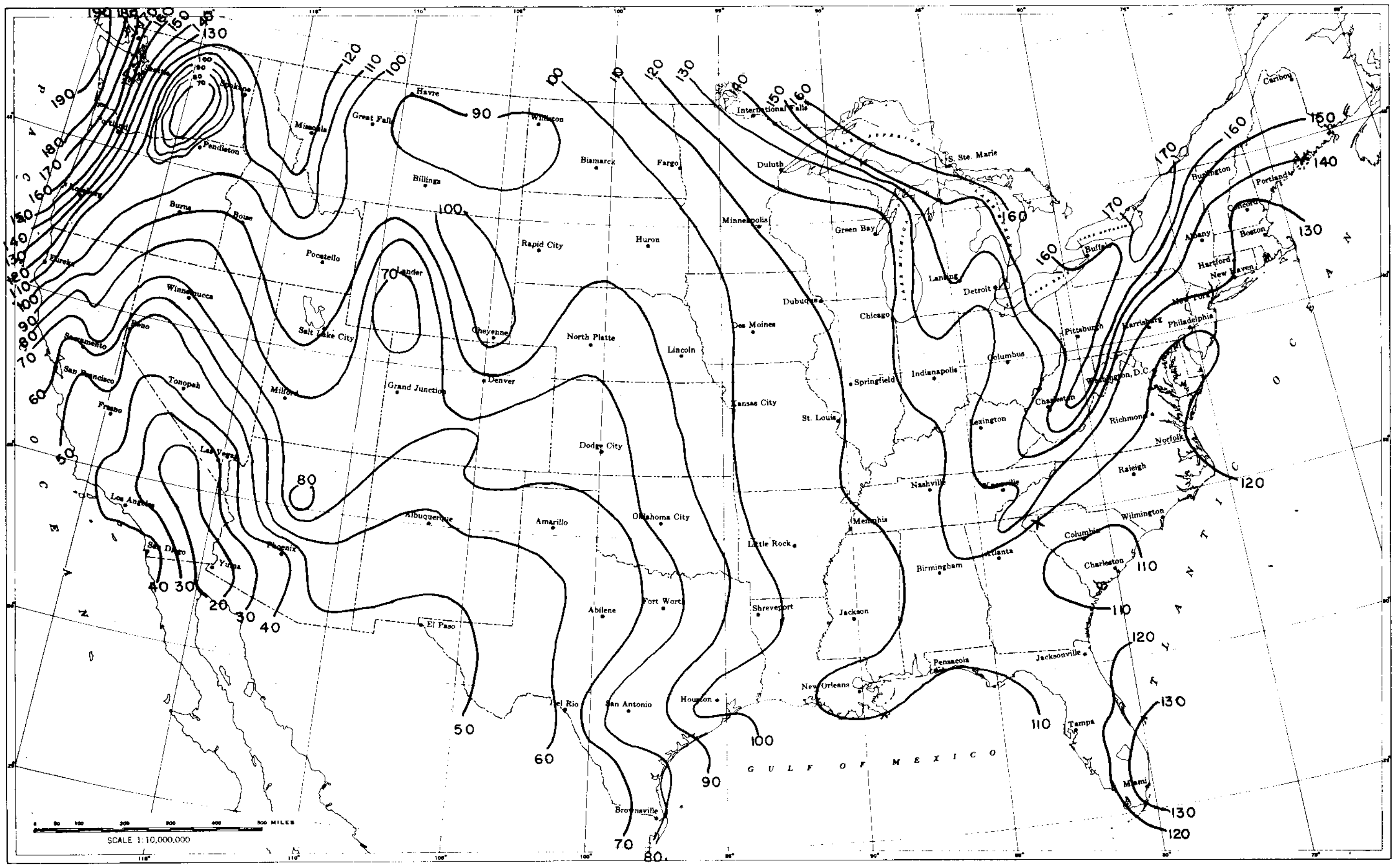

FIGURE 1.—Mean annual number of days with precipitation ≥0.01 inch (1929–58).

$$(n-1)s^2/\chi_2^2<\sigma^2<(n-1)s^2/\chi_1^2 \qquad (3)$$

where n, s, and σ are as before, and the two chi-square values χ_1^2 and χ_2^2 are 17.71 and 42.56 as selected for the 0.90 probability interval from a table of the chi-square distribution such as [6]. After insertion of the proper values and simplification, inequality (3) becomes

$$0.83s<\sigma<1.28s \qquad (4)$$

When necessary, inequality (4) was used in figure 2 in the same manner and for similar reasons that inequality (2) was used in figure 1.

4. USE OF CHARTS

A valuable characteristic of the Gaussian distribution is that it is defined completely by its mean and standard deviation, and it was for this reason that figures 1 and 2 were constructed. Interpolated values obtained from the charts can be used to make probability statements concerning the annual days with 0.01 inch or more of precipitation at any location covered by the charts.

For example, in figures 1 and 2 refer to the large "X" which marks the location of Clemson in the northwestern part of South Carolina. The mean interpolated from figure 1 is about 120 days, and the standard deviation from figure 2 is about 12.2 days. If it must be known, for instance, what the chances are of getting less than 108 days with measurable precipitation at Clemson in any one year, the expression $(x-\bar{x})/s$ is evaluated to obtain the entry value to any table of the normal distribution function, such as that contained in any statistics text or in [6]. In this case, with $x=108$, $\bar{x}=120$, and $s=12.2$, a slide rule calculation gives -0.98 standard deviation, which from [6] corresponds to a probability of 0.1635. Thus, at Clemson there is only a 16 percent chance of having less than 108 precipitation days in any one year; only about 1 year in 6 will have more than $365-108=257$ precipitation-free days; or the chances are about 5 to 1 that there will be more than 108 days with measurable precipitation in a year.

In the steeper mountain areas there are apt to be steep climatic gradients, as every meteorologist knows. Earlier in this paper, a statement was made concerning the conservative nature of the "day with 0.01 or more" statistic. While this is true when compared to some of the other climatic elements, this statistic is certainly not immune to the effects of rapid elevation changes within short horizontal distances. Consequently, only a general-

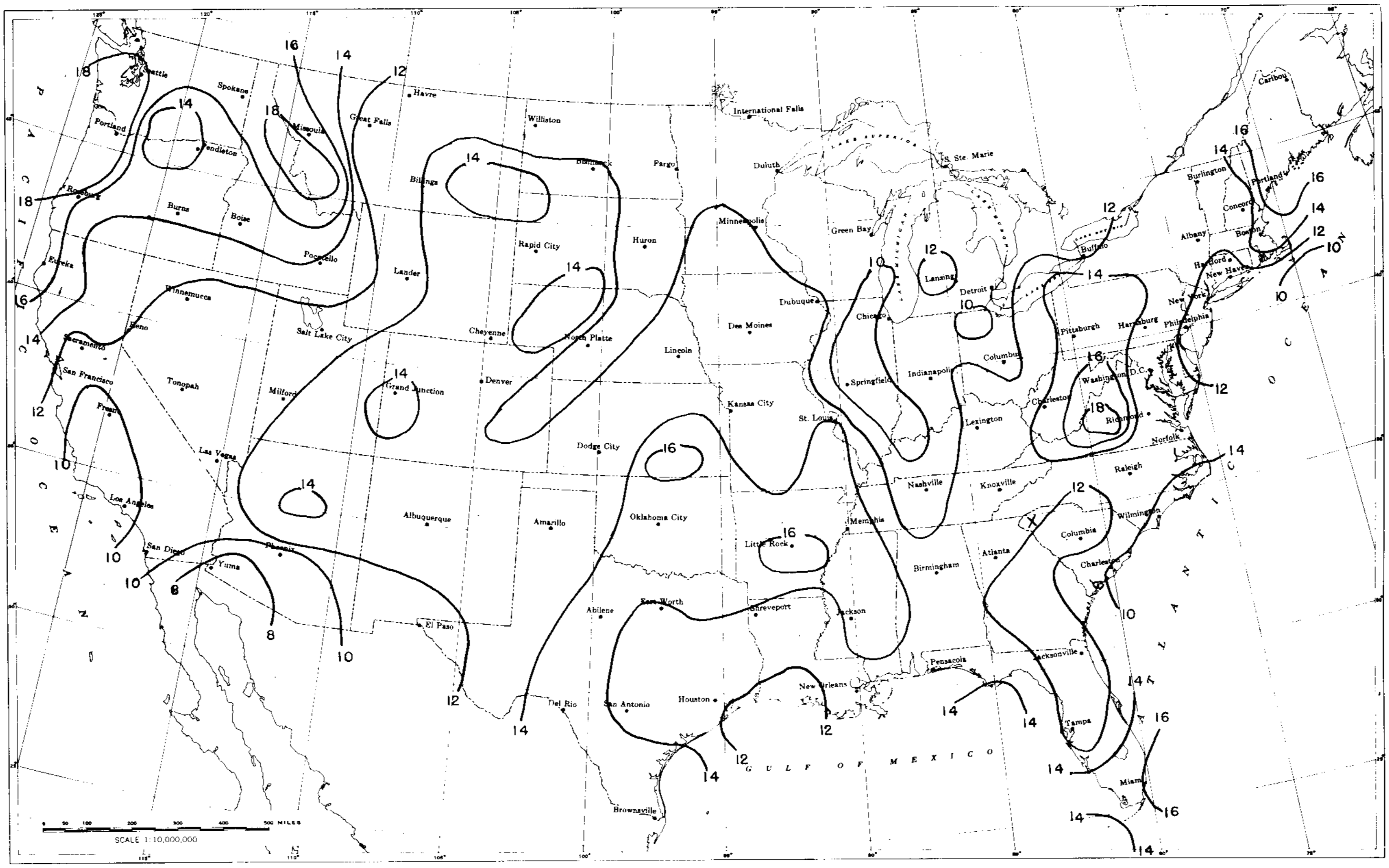

FIGURE 2.—Standard deviation of annual days with precipitation ≥0.01 inch (1929–58).

ized picture could be attempted in figures 1 and 2 over the major mountain areas of the country. For instance, mean values for Stampede Pass, Wash., and Mount Washington, N.H., are about 200 days, which is far above the expectancy shown in figure 1. These are extreme cases, however, and the suggested interpolation procedures should give good accuracy over most of the country.

5. THE SUBSTATION BIAS

Earlier, some introductory statements were made regarding the bias of cooperative station means toward low values as compared to first-order station means. This may be seen by examining figure 3. From data furnished by the Weather Bureau State Climatologist for Georgia, mean values and standard deviations for 45 substations were calculated and plotted on base maps which appear as figures 3 and 4. The isolines, however, were lifted directly from figures 1 and 2 so as to represent an analysis based entirely on first-order station data. Obviously, most substation means appear to be too small and there seems to be little uniformity as to the degree of bias. Furthermore, the substation standard deviations are frequently larger than they should be, showing a considerably greater data dispersion about the means.

Although figures 3 and 4 were based on a 30-year period (ending in 1952), the period is not uniform for each station because of missing record, nor is it the same as the 1929–58 period used for figures 1 and 2. These deficiencies could not be avoided without entailing considerably more data tabulation work, but this seemed unnecessary since the results confirmed the writer's earlier findings in 1953.

Calculations on actual 1929–58 data for Clemson, S.C. [9], produced an annual mean of 99.7 days with a standard deviation of 11.16. Use of these statistics, in the same manner given in the example earlier, gave rise to a 77 percent probability of having less than 108 days with measurable precipitation at Clemson, which value should be compared to the 16 percent probability found earlier. This indicates that, at least in some cases, the use of biased substation data results in less reliable probability statements than can be obtained through use of values interpolated from figures 1 and 2, as previously suggested.

Figure 5 is presented as further evidence of the substation bias. It appeared in [10], and was based on first-order station data as well as on data from several thousand substations. It should be compared with figure 1. Close

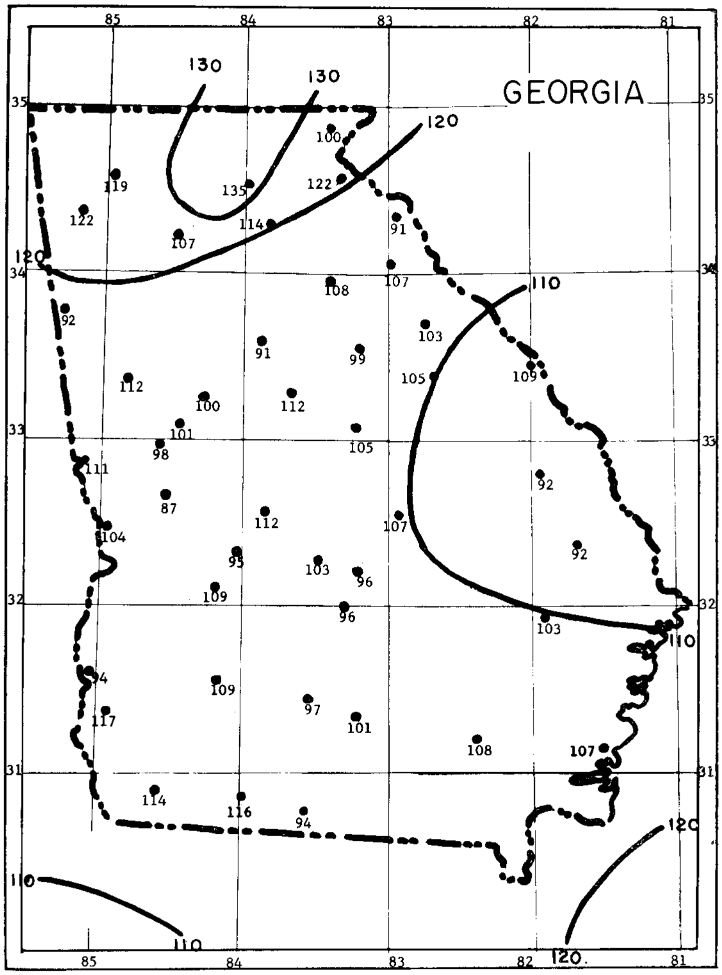

FIGURE 3.—Mean annual number of days with precipitation ≥0.01 inch.

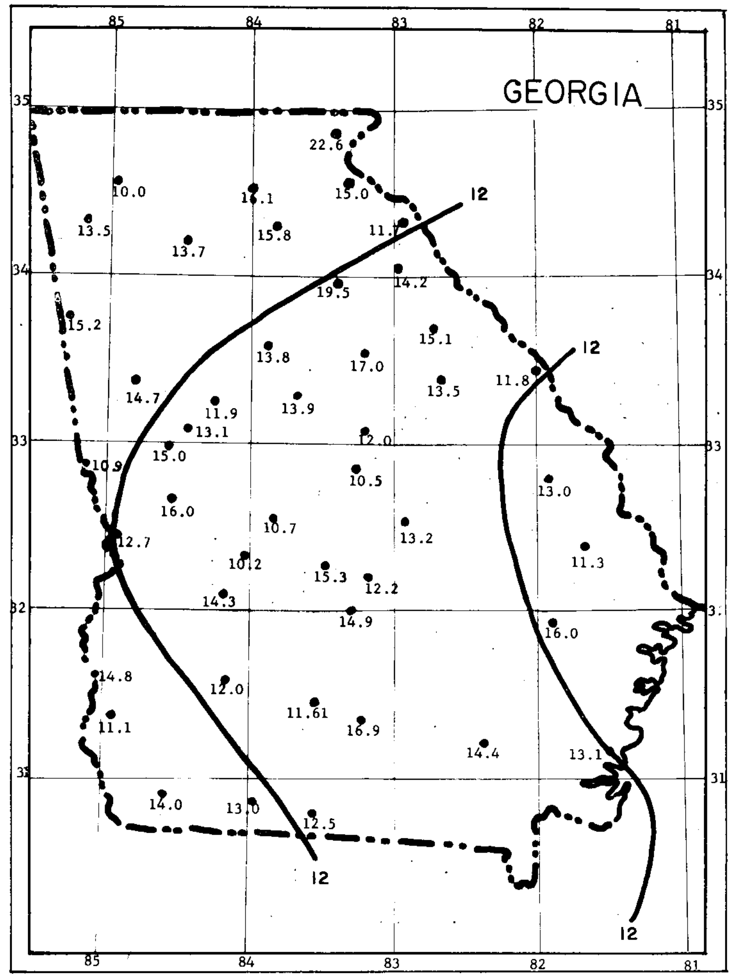

FIGURE 4.—Standard deviation of annual days with precipitation ≥0.01 inch.

inspection of figure 5 reveals the extreme irregularity of the isolines, which the writer regards as suspect, as well as many small closed contours or "islands" of high values. It turns out that many, if not most, of these islands surround a first-order station, and plotting these high-value islands gives a fairly good idea of the first-order station network used.

6. CAUSE OF SUBSTATION BIAS

In the absence of any real information on the substation bias cause, only conjectural statements can be made. To this writer, it seems plausible that the bias is a combination of several effects which probably differ from station to station and from time to time at the same station. Possibly some substation observers may not look in the rain gage each day, not realizing that it may have rained while they were engaged in their regular daily activities. In this manner, small amounts could be missed frequently.

Others, particularly those in the warmer areas, may find that small amounts have evaporated before they make their once-daily inspection of the gage. However, if this were a primary cause, the bias should follow a definite gradient, within an area, which should approximate the evaporation gradient. This does not seem to be the case in any data examined by the writer. For example, reference to Plate 1 of [11] shows that annual average evaporation increases from northern to southern Georgia, but the bias does not follow the same pattern, as will be seen from figure 3, even if changes in the mean values are taken into account as one goes from north to south.

7. FURTHER WORK

Although precipitation day statistics can be extremely useful in planning operations, it is not likely that the 0.01-inch threshold will be as useful as higher thresholds, e.g., 0.10, 0.25, 0.50, 1.00, etc., for most operations. For example, if a contractor bids on a dirt excavation job, he wants to know how many precipitation-days will be exceeded in a year with, say, a 75 percent probability so

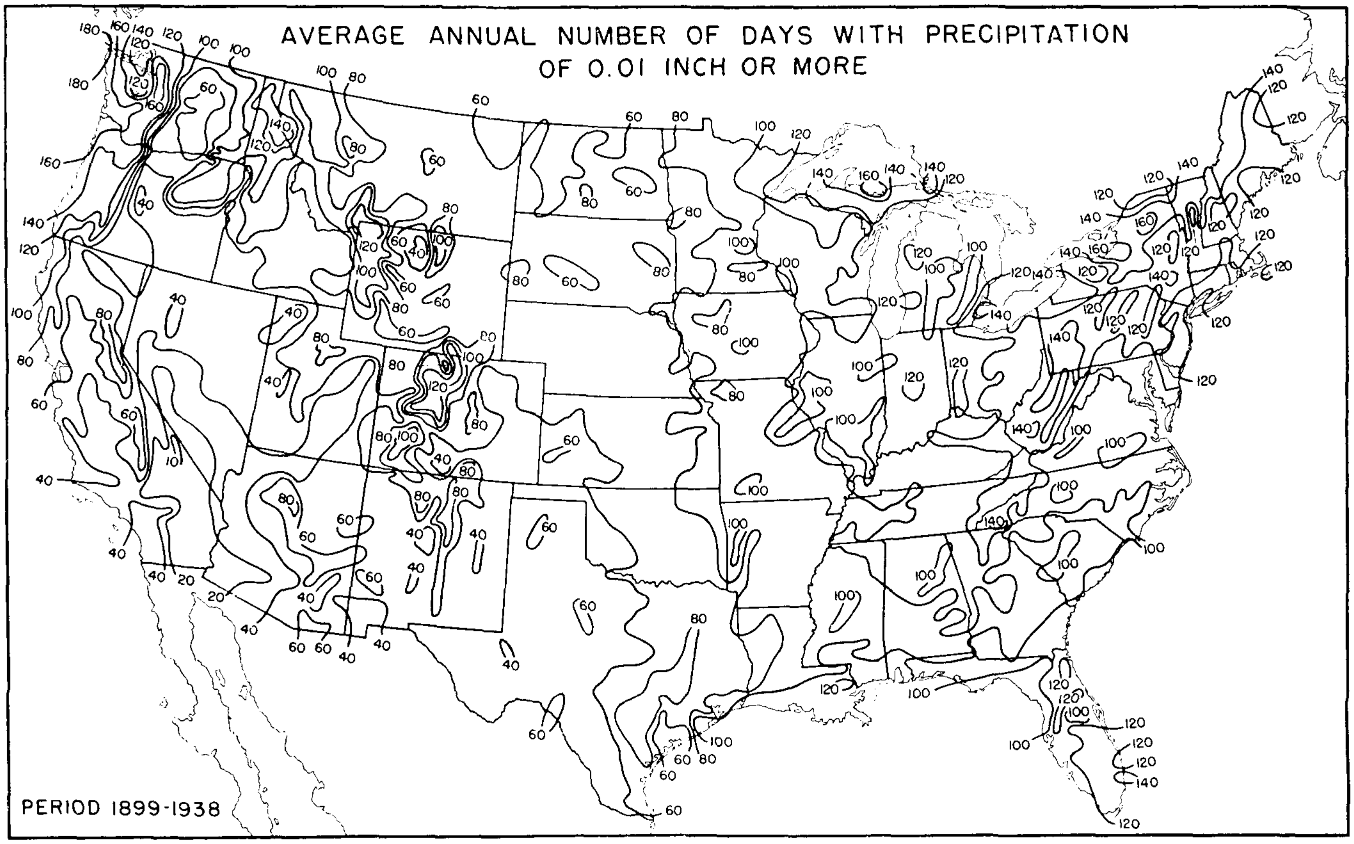

FIGURE 5.—Average annual number of days with precipitation of 0.01 inch or more, 1899–1938. (From [10].)

that he can calculate the probable job duration and the probable cost of idle equipment. If it would take 0.25 inch or more to halt his operations, then he should have available precipitation statistics based on that threshold value. If his operations were not planned to last a year, he would need similar statistics based on monthly or seasonal data. Therefore, monthly and annual charts of higher threshold values are needed.

Preliminary work by the writer indicates that the normal distribution should hold for higher thresholds up to at least 1.00 inch for annual as well as monthly precipitation data. At some point beyond 1.00 inch, it seems likely that the threshold occurrences, particularly monthly, begin to fall into the rare event class and will require different treatment. Investigation is continuing on this point.

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